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Garret N. Vanderplaats

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Garret N. Vanderplaats
Ames Research Center
Moffett Field, California

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NOTATION

A	airfoil enclosed area divided by c^2
a_i	participation coefficient
c	chord length
C_D	drag coefficient
C_L	lift coefficient
C_M	pitching-moment coefficient
C_P	pressure coefficient
$F(\bar{X})$	design objective function to be minimized or maximized
f	arbitrary function of $\bar{X}$
$G_j(\bar{X})$	constraint function
$[H]$	Hession matrix containing second partial derivatives
n	number of design variables
$\bar{S}^q$	search direction in optimization
t/c	thickness-to-chord ratio
$\bar{X}$	vector containing the design variables
X_i^{ℓ}	lower bound on design variable i
X_i^{u}	upper bound on design variable i
$\bar{Y}$	array of airfoil coordinates
Y^i	array of coordinates defining a shape function
α	airfoil angle of attack
α^*	move parameter in optimization
$\bar{\nabla}$	gradient operator
Δ	difference operator

Subscripts:

i	variable number
ℓs	lower surface
max	maximum
min	minimum
us	upper surface

Superscripts:

0	nominal design
q	iteration number
k	design number

APPROXIMATION CONCEPTS FOR NUMERICAL AIRFOIL OPTIMIZATION

Garret N. Vanderplaats

Ames Research Center

SUMMARY

An efficient algorithm for airfoil optimization is presented. The algorithm utilizes approximation concepts to reduce the number of aerodynamic analyses required to reach the optimum design. Examples are presented and compared with previous results. Optimization efficiency improvements of more than a factor of 2 are demonstrated. Much greater improvements in efficiency are demonstrated when analysis data obtained in previous designs are utilized. The method is a general optimization procedure and is not limited to this application. The method is intended for application to a wide range of engineering design problems.

INTRODUCTION

Numerical optimization techniques have been shown to provide a versatile tool for airfoil design. The usual approach has been to couple existing aerodynamic analysis codes with an optimization code to achieve the design capability. The primary effort has been directed toward application of these techniques to a wide variety of design problems while using increasingly sophisticated and time-consuming aerodynamic analysis programs.

The cost of this automated design, whereby a very time-consuming analysis program is executed repetitively (perhaps several hundred times), is necessarily an important consideration when judging the practicality of these techniques. Perhaps the simplest means of estimating cost is by the number of times the aerodynamic analysis program is executed during a design study. That is, for a given aerodynamics program, the cost of optimization is a direct function of the number of times the program is executed on the computer. Therefore, any improvement in optimization efficiency is directly measurable in design cost savings.

Very little effort has been directed toward improving the efficiency of the automated design process as applied to aerodynamic design. The principal improvement to date has been in the method of defining the airfoil. In references 1 and 2, polynomials were used to define the airfoil shape, with the coefficients of the polynomial being the design variables. In references 3 and 4, and in subsequent work, these polynomials were replaced by more general analytical or numerically defined shape functions. The result was an efficiency improvement of more than a factor of 2, together with improved airfoil

definition (ref. 3). However, efficiency improvements are still needed if numerical airfoil optimization is to become an economically feasible design approach when using sophisticated aerodynamic analysis codes.

The purpose here is to present a technique that improves the design efficiency by another factor of 2 or more. The basic approach is to develop approximations to the design problem using a minimal amount of information. The approximating functions are used in the optimization and the resulting design is analyzed precisely. This analysis information is added to the available data and the process is repeated until convergence to the optimum is achieved. (The idea of using approximation concepts in aerodynamic optimization originated from the observed success of similar techniques used by Schmit and Miura (ref. 5) in the field of structural optimization.)

To provide a background for the method, the basic concepts of numerical optimization are first presented. This is followed by a description of the present method of coupling an aerodynamic analysis program to an optimization program for automated design. The concept of optimization by sequential approximations is then presented, followed by a more precise mathematical formulation of the method and a summary of the design algorithm. Examples demonstrate the efficiency and reliability of the method and finally, some of the implications of this method for future development are discussed.

OPTIMIZATION CONCEPTS

Assume the airfoil is defined by the relationship

$$\bar{Y} = a_1 \bar{Y}^1 + a_2 \bar{Y}^2 + \ldots + a_n \bar{Y}^n \tag{1}$$

where $\bar{Y}$ is a vector containing the upper and lower coordinates of the airfoil and $\bar{Y}^i$ are shape functions that may themselves define airfoils. The coefficients $a_1, a_2 \ldots, a_n$ are referred to as participation coefficients. Now assume it is desired to find the airfoil that minimizes the drag coefficient C_D with constraints on lift coefficient C_L, thickness-to-chord ratio, t/c, etc., at a specified Mach number and angle of attack. The participation coefficients $a_1 - a_n$ are the design variables, and will be changed during the optimization process. The n-dimensional space spanned by the design variables is called the design space.

The optimization problem can be stated mathematically as:

Minimize

$$C_D \tag{2}$$

subject to

$$C_L \geq C_{L_{min}} \tag{3}$$

$$(t/c) \geq (t/c)_{min} \tag{4}$$

where C_D, C_L, and t/c are functions of $a_1, a_2, \ldots, a_n$. This can be generalized to be:

Minimize

$$F(\bar{X}) \tag{5}$$

subject to

$$G_j(\bar{X}) \leq 0 \qquad j = 1, m \tag{6}$$

$$X_i^{\ell} \leq X_i \leq X_i^{u} \quad i = 1, n \tag{7}$$

where $\bar{X}$ is a vector containing the design variables, a_i, $i = 1,n$. There are a total of m constraints. The lift constraint of equation (3) is written in the form of equation (6) as

$$1 - \frac{C_L}{C_{L_{min}}} \leq 0 \tag{8}$$

Similarly, from equation (4)

$$1 - \frac{(t/c)}{(t/c)_{min}} \leq 0 \tag{9}$$

The parameters X_i^{ℓ} and X_i^{u} of equation (7) are referred to as side constraints that limit the region of search for the optimum. Although side constraints could be included in equation (6), they are usually treated separately for convenience and efficiency.

The optimization problem of equations (5)-(7) is quite general. If it is desired to maximize C_L with a constraint on C_D, $-C_L$ is minimized. Also, the constraint set of equation (6) is not limited to constraints at the design flight condition. With this formulation, the airfoil can be designed at a given flight condition with constraints at other flight conditions so long as the appropriate information is calculated during the aerodynamic analysis. If the inequality conditions of equations (6) and (7) are satisfied, the design is said to be feasible. If any of these conditions are violated, the design is called infeasible.

The optimization process typically proceeds in an iterative fashion as:

$$\bar{X}^q = \bar{X}^{q-1} + \alpha^* \bar{S}^q \tag{10}$$

An initial design, $\bar{X}^0$, must be provided which may or may not define a feasible design. The superscript q is the iteration number. Vector $\bar{S}^q$ is the

search direction and α^* is a scaler determining the move distance in direction $\bar{S}^q$. The notation α is used for consistency with mathematical programming literature and should not be confused with the airfoil angle of attack.

If gradient methods are used, the optimization process consists of two steps. The first is determination of a move direction $\bar{S}^q$ that will improve the design without violating any constraints; the second is calculation of α^* such that the objective is reduced as much as possible in this direction.

This may be understood by considering a two-variable design problem where C_D is minimized subject to constraints on C_L and t/c. A hypothetical problem is shown in figure 1 which shows contours of constant drag and shows the C_L and t/c constraint boundaries. Assume an initial design is given at point A, with no active or violated constraints. Using gradient methods, the process begins by perturbing each component of $\bar{X}$ to determine its effect on the objective. That is, the gradient of C_D is calculated by finite difference using a single forward step, and the gradient vector is constructed as

$$\bar{\nabla}F = \bar{\nabla}C_D = \begin{Bmatrix} \frac{\partial C_D}{\partial X_1} \\ \frac{\partial C_D}{\partial X_2} \\ \vdots \\ \frac{\partial C_D}{\partial X_n} \end{Bmatrix} \simeq \begin{Bmatrix} \frac{\Delta C_D}{\Delta X_1} \\ \frac{\Delta C_D}{\Delta X_2} \\ \vdots \\ \frac{\Delta C_D}{\Delta X_n} \end{Bmatrix} \tag{11}$$

It is obvious that the greatest improvement of the objective function is achieved by moving in the negative gradient, or steepest descent direction, so that $\bar{S} = -\bar{\nabla}C_D$. Knowing the search direction, $\bar{S}^1$, the scalar α^* that will minimize C_D in this direction must be found. This is a one-variable minimization problem. Several somewhat arbitrary values of α are defined and the airfoil is analyzed at each point, $\bar{X} = \bar{X}^0 + \alpha\bar{S}^1$. A polynomial is usually fit to these points and a more precise $\alpha = \alpha^*$ is calculated at point B in figure 1, ending the first optimization iteration. The second iteration begins by again perturbing the design variables to obtain the gradient of the objective. Using the conjugate direction algorithm of Fletcher and Reeves (ref. 6) a new search direction, $\bar{S}^2$, is found which will again reduce the objective. A search is performed in this direction, leading to point C, completing iteration two. At C, the lift constraint is active ($G_j = 0$) and a direction is found that will reduce C_D without violating this constraint. The gradient of both the objective and active constraint are calculated and a new search direction, $\bar{S}^3$, is found using Zoutendijk's method of feasible

directions (refs. 7,8). The process is repeated until a design at E is obtained where no direction can be found that will reduce the objective without violating the constraints and this design is called optimum. Logic is included in the algorithm so that if an initial design is defined at point F, the constraint violations are overcome to yield a final design at point E.

The optimization procedure described above is essentially that used in the CONMIN program (ref. 9). In a typical design, about 10 iterations are required to achieve an optimum design. For each iteration, n aerodynamic analyses are used to calculate the required gradient information by finite difference. To determine α^* requires an average of three analyses so that a total of $10(n + 3) = 10n + 30$ aerodynamic analyses are required for optimization for a single flight condition. Although quite efficient from an optimization viewpoint, that many executions of a sophisticated aerodynamics program can be very expensive. Therefore, it is desirable to reduce the number of required analyses as much as possible. This improvement in optimization efficiency is the subject here. The technique will be developed by first reviewing the approach currently used for aerodynamic optimization.

Previous Method of Aerodynamic Optimization

At the present time, most airfoil optimization is performed by coupling the aerodynamics program to the optimization program as shown in figure 2. Each time the optimization program defines a new design, either for finite difference gradient computations or for determining α^*, the aerodynamics program is called for a complete analysis. For the example of figure 1, a set of analyses is performed as indicated in figure 3. During optimization, at iteration j, very little information from previous iterations is used. At point B in figure 1, the vector $\bar{S}^1$ is used to calculate $\bar{S}^2$ so that, if no constraints are active, prior information is used. However, if one or more constraints are active or violated (the usual situation), no prior information is used.

It can be argued intuitively that all calculated information should be of value in guiding the optimization process. Furthermore, in a design study, numerous optimizations are usually performed. For example, one optimization may be done to minimize C_D with constraints on C_L and, later, another optimization done to minimize C_M with constraints on C_L and C_D. It may be expected that, because many airfoils were analyzed during the first optimization, a second optimization at the same flight condition should utilize this available information. One way to do this would be to approximate the required functions using available information. This would provide explicit functions which could now be optimized independent of the time consuming aerodynamic analysis program. Aerodynamic analysis is still used to improve the approximation, leading to a precise solution.

The general procedure is outlined in the following section.

Optimization by Sequential Approximations

Assume it is desired to approximate the aerodynamic parameters linearly in terms of the design variables, $\bar{X}$. Then, for example,

$$C_D \approx C_D{}^0 + \frac{\partial C_D}{\partial X_1}(X_1 - X_1{}^0) + \frac{\partial C_D}{\partial X_2}(X_2 - X_2{}^0) + \ldots + \frac{\partial C_D}{\partial X_n}(X_n - X_n{}^0) \quad (12)$$

$$= C_D{}^0 + \bar{\nabla} C_D \cdot (\bar{X} - \bar{X}^0) \quad (13)$$

where the superscript denotes the point about which the curve fit was done. Now at point A in figure 1, this information is obtained by $n + 1$ analyses; an initial function evaluation plus finite difference gradients. Similar information can be simultaneously obtained for C_L and t/c. This provides explicit, but approximate, expressions for the functions. The two-variable representation for this linearized problem is shown in figure 4, where the objective and constraints are now linear functions of the design variables. It is clear from figure 4 that the solution to the approximate optimization problem is unbounded so that this would not yield meaningful results. However, by limiting the design change to some reasonable bounds, shown by the rectangle, the optimization will have a solution. At that point, the problem could be linearized again and resolved. Note that at the end of this first optimization the airfoil will be analyzed precisely and the results compared with the approximate solution. Therefore, $n + 2$ analyses are now available to set up a new approximate problem. The problem could be linearized by either using only $n + 1$ of these analyses or using a least squares fit to all of the data.

Note that, because this approximate problem is strictly linear, linear programming techniques could be used (Simplex method, ref. 10). The technique of repeatedly linearizing a nonlinear problem and solving with linear programming is known as "sequential linear programming" and has been used with success in structural design (ref. 11).

An alternative approach to sequential linear programming, and the one used here, is to use the excess data to develop higher order approximations to the functions. Then, in this case, the extra analysis would be used to provide a second-order approximation with respect to X_1. This new approximate problem is optimized, followed by a new precise analysis and the process repeated until the solution has converged. When a total of $1 + n + n(n + 1)/2$ analyses are available, a full quadratic approximation is possible. Subsequent to this, new analyses are used in a weighted least squares fit, rather than obtaining ever higher order approximations. Only a second-order approximation is used because higher order approximations (1) would require excessive data, (2) tend to model noise in the data, and (3) have been found to be unnecessary.

If a quadratic design problem were being considered, the quadratic approximation would be precise. If the problem at hand can be approximated closely by a quadratic function, this method can be expected to be

competitive. Assuming only a few analyses are required beyond that necessary for a quadratic approximation, this method is competitive for problems of fewer than 20 design variables, and is twice as efficient for problems of 10 variables. More dramatic improvements are realized by using analysis data obtained in one optimization as data for subsequent optimizations.

The concept of sequential approximations is shown in block diagram form in figure 5. Note, in comparison to figure 2, that the optimization program never directly works with the aerodynamic analysis, but only optimizes the approximating functions. Because the evaluation of the approximate functions is short and is explicit, the efficiency of the optimization code itself is of minor importance because the necessary function evaluations are quite rapid. Also, gradients of the approximating functions are easily calculated analytically, a feature that improves the efficiency of the approximate optimization.

In the following section, the sequential approximation approach to optimization is outlined mathematically.

Mathematical Formulation

Consider the Taylor series expansion of any function:

$$f \simeq f^0 + \bar{\nabla} f \cdot (\bar{X} - \bar{X}^0) + \frac{1}{2} (\bar{X} - \bar{X}^0) \cdot H \cdot (\bar{X} - \bar{X}^0) + \cdots \tag{14}$$

In equation (14), $\bar{X}^0$ is the point about which the expansion is being performed, f^0 is the corresponding function value, and $\bar{\nabla} f$ is the vector of first partial derivatives (gradient). The matrix of second partial derivatives (Hession matrix), H, is symmetric:

$$\bar{\nabla} f = \begin{Bmatrix} \frac{\partial f}{\partial X_1} \\ \frac{\partial f}{\partial X_2} \\ \vdots \\ \frac{\partial f}{\partial X_n} \end{Bmatrix} \qquad H = \begin{Bmatrix} \frac{\partial^2 f}{\partial X_1^2} & \frac{\partial^2 f}{\partial X_1 \partial X_2} & \cdots & \frac{\partial^2 f}{\partial X_1 \partial X_n} \\ & \frac{\partial^2 f}{\partial X_2^2} & & \\ & & \ddots & \vdots \\ \text{SYMMETRIC} & & & \frac{\partial^2 f}{\partial X_n^2} \end{Bmatrix} \tag{15}$$

In aerodynamic optimization, this information is not usually available analytically, but can be calculated numerically. The usual approach is to use finite difference gradients to provide a good approximation to $\bar{\nabla} f$ and H. This assumes a small finite difference step size to insure that the

approximation is good. However, it may be desirable to utilize previously calculated data to determine the components of $\bar{\nabla}f$ and H, recognizing that the accuracy of this approximation will not be as great as if small finite difference steps were used.

The terms up through second order of equation (14) may be written in more compact form as

$$\Delta f = \bar{\nabla}f \cdot \Delta\bar{X} + \frac{1}{2}\,\Delta\bar{X}^T\,[H]\,\Delta\bar{X} \tag{16}$$

where

$$\Delta\bar{X} = \bar{X} - \bar{X}^0$$

and

$$\Delta f = f - f^0$$

Equation (16) may be expanded as

$$\begin{aligned}\Delta f = {} & \nabla f_1\,\Delta X_1 + \nabla f_2 \Delta X_2 + \cdots \nabla f_n\,\Delta X_n \\ & + \frac{1}{2}\,(H_{11}\,\Delta X_1^{\,2} + H_{22}\,\Delta X_2^{\,2} + \cdots + H_{nn}\,\Delta X_n^{\,2}) \\ & + H_{12}\Delta X_1\Delta X_2 + H_{13}\Delta X_1\Delta X_3 + \cdots + H_{1n}\Delta X_1\Delta X_n \\ & + H_{23}\Delta X_2\Delta X_3 + \cdots H_{n-1,n}\,\Delta X_{n-1}\,\Delta X_n\end{aligned} \tag{17}$$

Now assume a nominal design, $\bar{X}^0$, has been analyzed to yield f^0. Also, numerous other designs, $\bar{X}^1, \ldots \bar{X}^k$ have been analyzed to give $f^1, \ldots, f^k$. Let

$$\Delta\bar{X}^i = \bar{X}^i - \bar{X}^0 \qquad i = 1, \ldots, k$$

and

$$\Delta f^i = f^i - f^0 \qquad i = 1, \ldots, k$$

Then we can write k equations of the form (17). The unknowns are $\nabla f_1 \ldots \nabla f_n$ and $H_{11}, H_{12} \ldots H_{nn}$ for a total of $\ell = n + n(n+1)/2$ unknowns. Remembering that one analysis was required for the nominal design, $\bar{X}^0$, a total of $\ell + 1$ designs are required. Thus, if $k \geq \ell + 1$, the unknowns can be determined. Equation (17) is linear in the unknown coefficients. Writing this equation for each of ℓ designs yields ℓ equations which can be solved directly.

If more than ℓ designs are available, a weighted least squares fit is used. If less than ℓ designs are used, fewer coefficients are calculated.

In the extreme, if only the nominal design and one other design are available, only the first term in equation (17) is calculated. This approximation can be used to optimize with respect to variable X_1 only. The result of that optimization is then analyzed and used to calculate the first two terms in equation (17). Optimization can then be performed with respect to X_1 and X_2, and so on. In this fashion, all data are used to guide the optimization, and the design is continually improving. Figure 6 depicts the sequence of designs that may be analyzed precisely in the two variable example.

Design Algorithm

Given the capability of developing the approximate Taylor series expansion of the various aerodynamic and geometric functions, it is incorporated into an optimization algorithm as follows:

1. Given k initial designs, $k \geq 2$
2. Create the Taylor series expansion about the current "best" design
3. Number of design variables, NDV = MIN(k,n)
4. Set limits on the design variables, say $\bar{X}^{\ell} = 0.8*\bar{X}^{0}$ and $\bar{X}^{u} = 1.2*X^{0}$
5. Optimize the approximating functions
6. Analyze the proposed optimum
7. Add results to data set; set $k = k + 1$
8. If $k \leq n + 1$ go to step 2
9. Check convergence
10. If satisfied, print final results; otherwise, go to step 2

A FORTRAN computer code was written for this technique and a block diagram of the major operations is shown in figure 5.

The CONMIN program (ref. 9) was used for the optimization capability.

In the following section, design examples are presented to demonstrate the efficiency of the method.

DESIGN EXAMPLES

Examples are presented here to identify the generality and efficiency of approximation concepts as applied to airfoil optimization. Four existing

airfoils are used as the design basis vectors in equation (1). These are the NACA 2412, NACA 64_1-412, NACA 65_2-415, and the NACA 64_2-A215 airfoils. The coordinates are defined at 50 points along the upper and lower surfaces. The coordinates are approximate, obtained from curve fits of the existing airfoil data (refs. 12, 13) and no attempt was made to precisely match the data given in the references. Two additional basis vectors were used to impose the boundary conditions at the trailing edge of the airfoil: $Y_{us} = X/C$, $Y_{\ell s} = 0$ and $Y_{us} = 0$, $Y_{\ell s} = -(X/C)$. The shapes defined by these six basis vectors are shown in figure 7. These basis vectors are the same as those used in reference 3. For consistency, the same aerodynamic analysis code (ref. 14) was also used. Three of the design examples of reference 3 are solved here, two of which were also presented in reference 2.

Example 1: Lift Maximization, $M = 0.1$, $\alpha = 6°$

Figure 8 shows the results of optimization of an airfoil for maximum lift. The design constraints are listed on the figure and are the same as Case 2 of reference 3. Additional numerical results are given in tables 1 and 2. This optimization required 19 aerodynamic analyses; 44 were required previously. Although it may be argued that this airfoil is impractical, it must be remembered that this airfoil mathematically satisfies the design constraints. Also, the lift coefficient obtained here, $C_L = 1.144$, is better than the one obtained before, $C_L = 1.108$. The fact that this airfoil was not obtained using the method of reference 3 suggests that the present method is numerically better conditioned for optimization. Furthermore, these results were obtained using less than half the number of aerodynamic analyses used previously.

The quality of the approximation to the lift coefficient may be judged from figure 9. Because there are four independent design variables, the full second-order Taylor series expansion of the functions requires 15 analyses. It is intriguing to note that on the sixteenth analysis and beyond, the approximation for this case is quite precise.

Example 2: Lift Maximization, $M = 0.75$, $\alpha = 0°$

Although optimization using sequential approximations works well for low-speed airfoils, it might be expected that the technique would not be adequate for high-speed applications where the nature of the flow field about the airfoil can be quite sensitive to small changes in the airfoil shape. To study this possibility, Case 3 of reference 3 was solved using the present method. Here, the lift coefficient was maximized subject to a constraint on wave drag. A value of $C_L = 0.4211$ was obtained after 27 analyses, compared to $C_L = 0.4188$ obtained in 70 analyses previously. The results are shown as example 2A in figure 10. Using the present method, the optimization continued to mathematically improve the airfoil. After 37 analyses, the airfoil of figure 11 (example 2B) was obtained. The optimization terminated after 48 analyses, yielding the airfoil shown in figure 12 (example 2C). Note the significant changes in pressure distribution among

figures 10-13. The comparison of approximate and precise values of C_L and C_D are shown in figure 13. The two values agree well at 27 analyses. After 37 analyses the flow field is as shown in figure 11. The comparison between approximate and precise values is then poor until the convergence to the final optimum after 48 analyses. The disagreement between approximate and precise function values is understandable from figure 14 which shows the airfoil corresponding to the forty-third analysis. Note the reverse curvature of the upper surface near the leading edge and near the 60-percent chord. This results from the inability to properly model a supercritical airfoil using the NACA basis airfoils. The optimization was able to effectively utilize these data to redirect the optimization process, leading to the final converged solution. This final airfoil, shown in figure 12, has the same characteristics as the airfoil of figure 14, but to a lesser degree. In a practical design situation, it would be desirable at this point to add other basis vectors that represent supercritical airfoils, remembering that the 48 analyses already obtained provide useful data for the expanded optimization. The results of this optimization are given in tabular form in tables 1 and 2.

Example 3: Wave Drag Minimization, $M = 0.75$, $\alpha = 0°$

To demonstrate the efficiency of the present method when multiple optimizations at the same flight condition are performed, Case 4 of reference 3 was solved. The 48 analyses performed to solve example 2 here were used as initial data. In reference 3, the optimum airfoil from the previous design was used as a starting point for this design. In the present study, the twenty-seventh analysis (fig. 10) was used as the nominal design about which the first Taylor series expansion was performed. An optimum design of $C_D = 0.0009$ was obtained using only two additional analyses. The resulting airfoil is given as example 3A in figure 15 and in tables 1 and 2. This result compares to an optimum $C_D = 0.0007$ obtained previously using 44 aerodynamic analyses.

As an additional exercise, this design was repeated beginning with the forty-eighth analysis of example 2 as the initial nominal airfoil. An optimum $C_D = 0.0003$ was obtained using four additional analyses. This design (example 3B) is presented in figure 16 and in tables 1 and 2. As seen from the figures, examples 3A and 3B represent quite different airfoils, although the actual calculated wave drag is negligible in each case.

DISCUSSION

Approximation concepts as applied to aerodynamic design have been presented. The technique has been shown to be more versatile and efficient than earlier techniques. It is not limited to two-dimensional airfoils, or to a single flight condition, and it is not limited to the aerodynamic analysis code used to provide the examples. The technique is a general

automated design procedure that may be applied to a wide variety of engineering design problems in addition to the one considered here.

The results presented here are considered preliminary and, as experience is gained through application of the method, further refinements can be expected.

Future effort will concentrate on development of the computer code, written as part of this study, into a generally available code for distribution, applicable to problems of broad engineering interest. Efforts in aerodynamic design will be directed toward extension to more general design situations. Of fundamental importance is development of data storage and retrieval systems so that the ever increasing body of available aerodynamic data can be easily utilized in design. Finally, effort will be directed toward the use of experimental data as a basis for design. The general goal is to develop a distributable computer program and data base that the user can apply to his particular design problem at extremely low cost.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, California 94035, November 17, 1978

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TABLE 1.- DESIGN INFORMATION

Example	Comment	Coefficients			
		a_1	a_2	a_3	a_4
1	Initial design	1.0	0	0	0
1	Optimum design	4.9323	-0.6653	-0.4802	-2.3018
2,3	Initial design	1.0	0	0	0
2A	After 27 analyses	.4989	.4004	-.5997	.6181
2B	After 37 analyses	.0533	.9713	-1.3342	1.2632
2C	Optimum design	-.4518	1.3518	-1.8513	1.8959
2D	Forty-third analysis	-.4954	1.2148	-1.8393	2.0302
3A	Initial analysis no. 27	.4205	.1930	-.6256	.8698
3B	Initial analysis no. 48	-.2137	-.7223	-1.3723	1.7285

TABLE 2.- ANALYSIS INFORMATION

Example	C_L	C_{D_w}	C_M	A[a]	t/c	Camber	Number of analyses		
							Ref. 2	Ref. 3	New method
1 init.	0.9864	0	-0.0645	0.0809	0.1193	0.0201	-	-	-
1 opt.	1.1437	0	-.7496	.0750	.1501	.0298	103	44	19
2,3 init.	.4824	.0113	-.1068	.0821	.1201	.0201	-	-	-
2A	.4211	.0039	-.1017	.0750	.1119	.0101	143	42	27
2B	.4702	.0039	-.1341	.0750	.1154	.0108	143	70	37
2C	.4716	.0040	-.1505	.0755	.1191	.0117	143	70	48
2D	.4024	.0019	-.1319	.0759	.1191	.0124	-	-	-
3A	.3057	.0009	-.0770	.0751	.1108	.0077	-	44	2
3B	.3009	.0003	-.0945	.0765	.1168	.0079	-	44	4

[a]Cross-sectional area divided by the chord squared.

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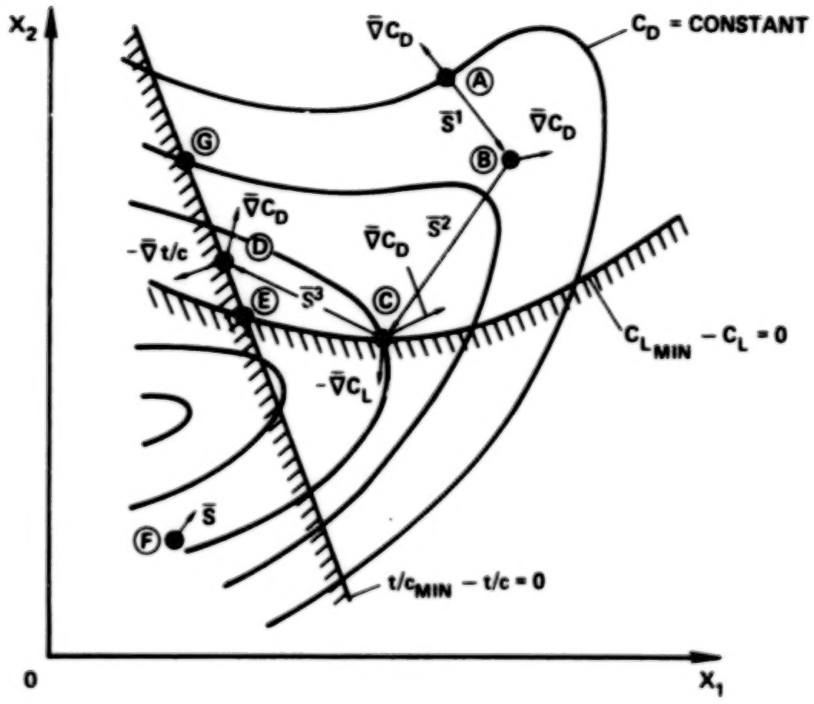


Figure 1.— Two-variable design space.

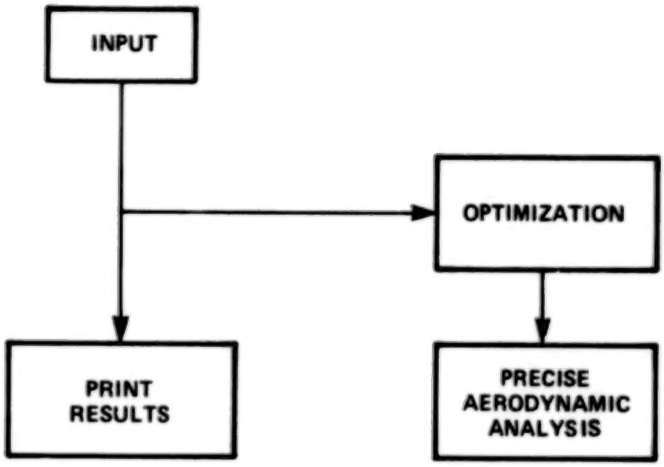


Figure 2.— Previous program organization.

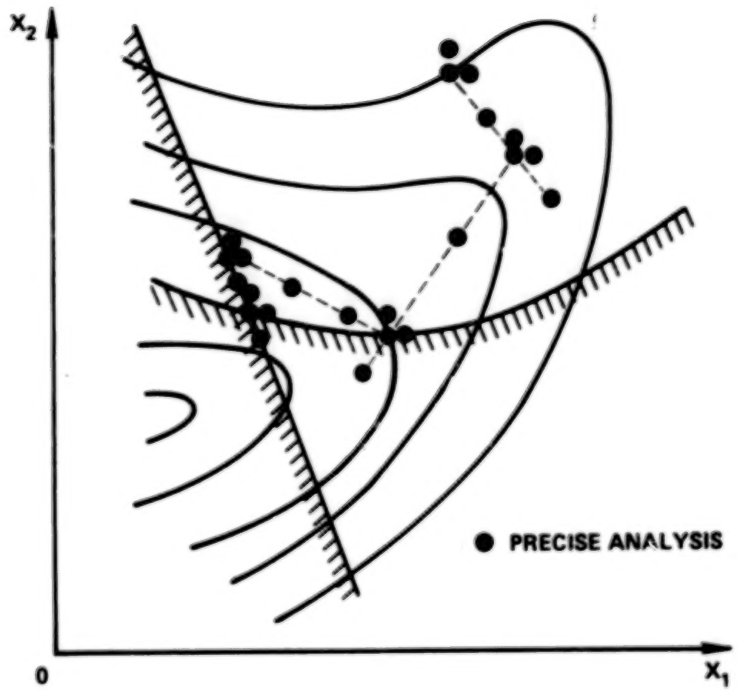


Figure 3.— Sequence of precise analyses: previous method.

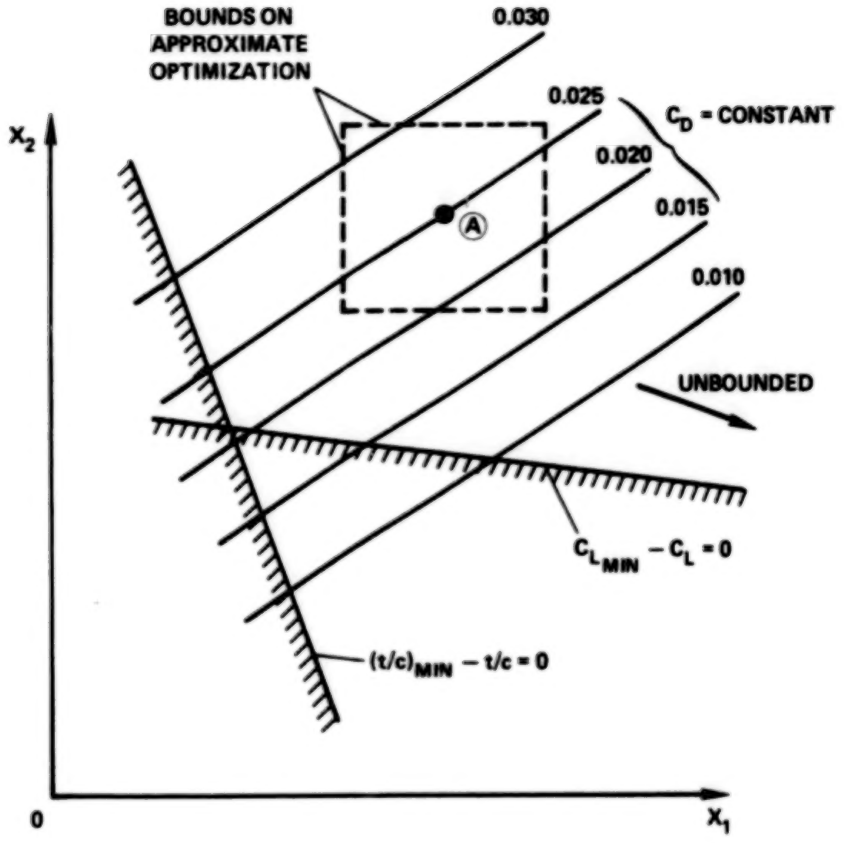


Figure 4.— Linear approximation to initial design.

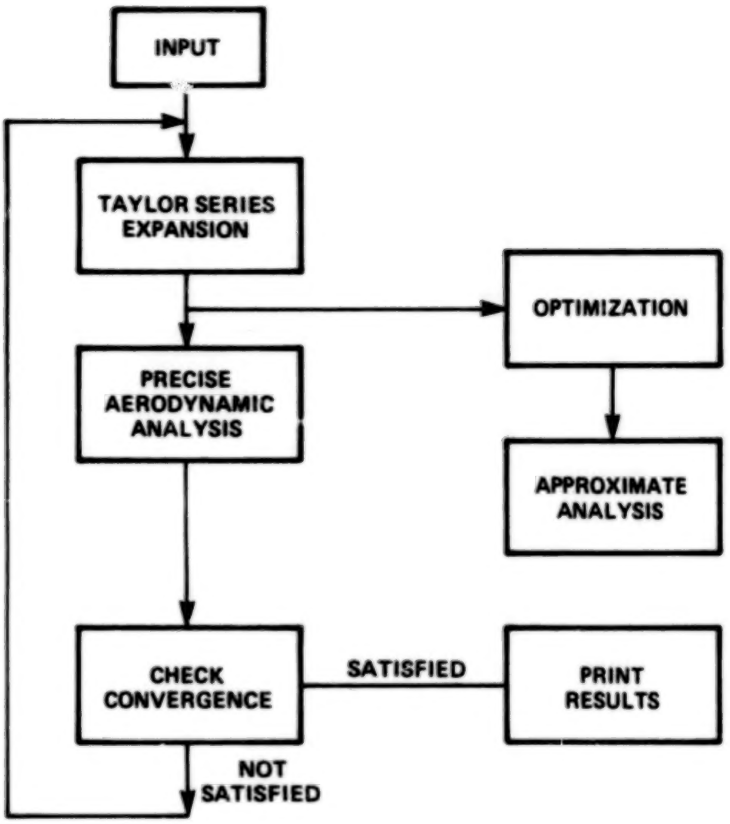


Figure 5.— New program organization.

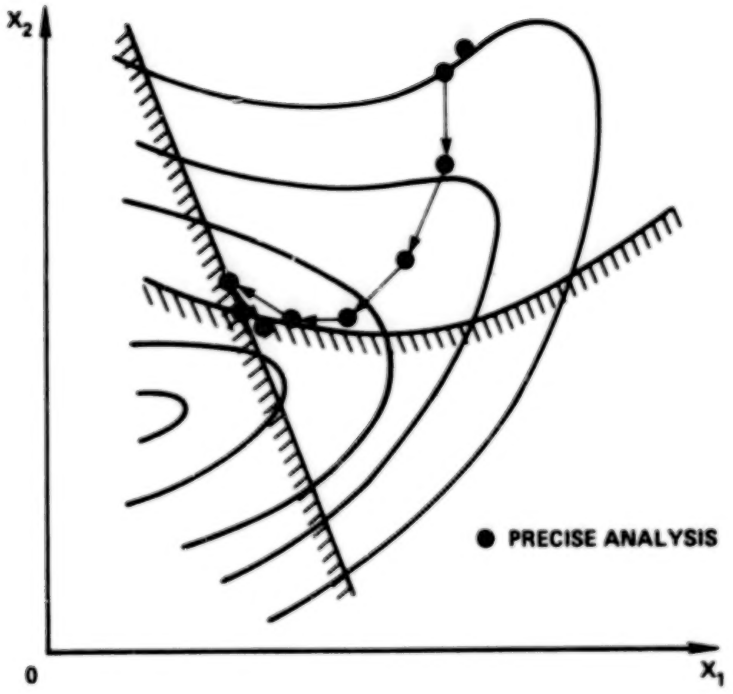
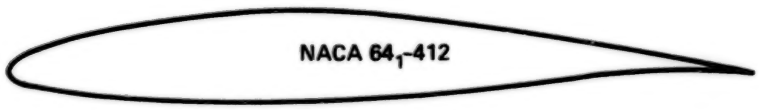
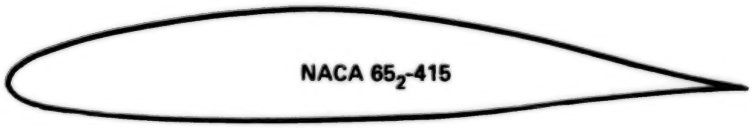


Figure 6.— Sequence of precise analyses using approximation techniques.

BASIS SHAPE 1



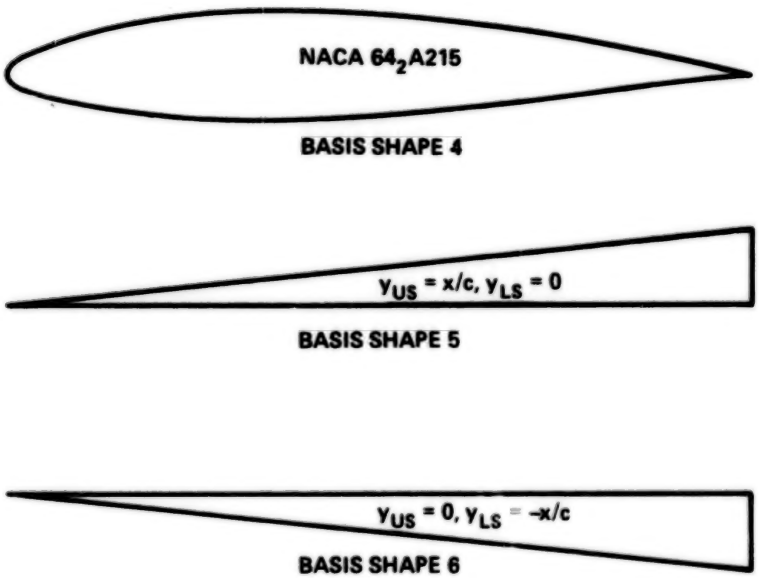
BASIS SHAPE 2



BASIS SHAPE 3

(a) Shapes 1 through 3.

Figure 7.— Basis shapes.



(b) Shapes 4 through 6.

Figure 7.— Concluded.

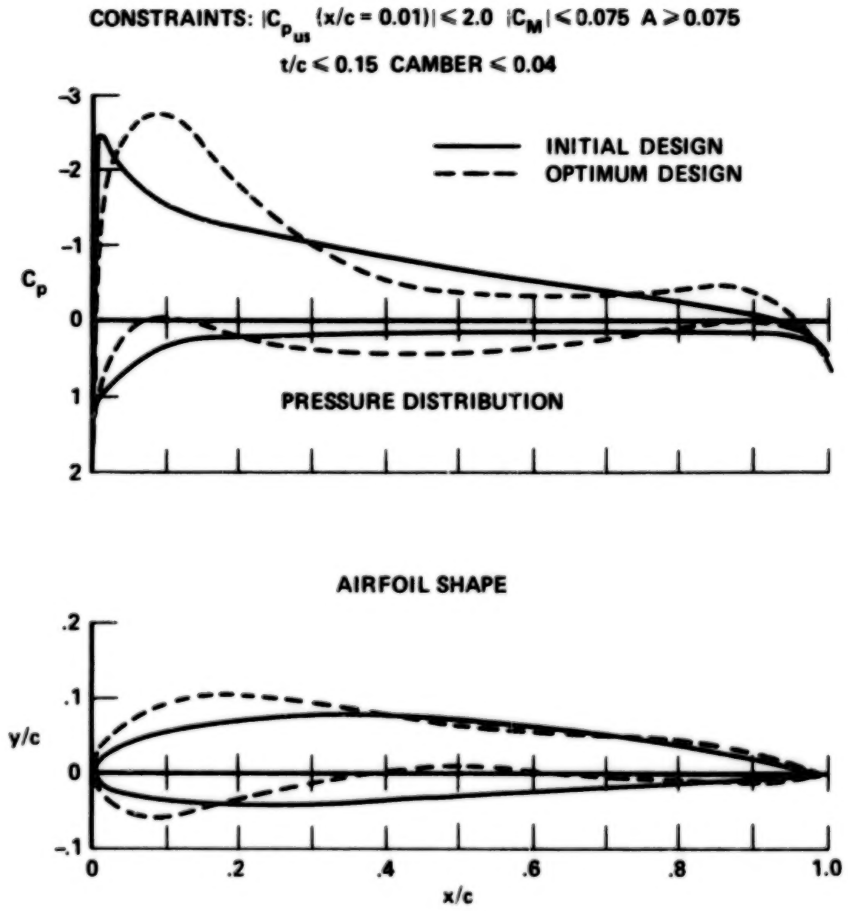


Figure 8.— Example 1: Lift maximization, M = 0.1, α = 6°.

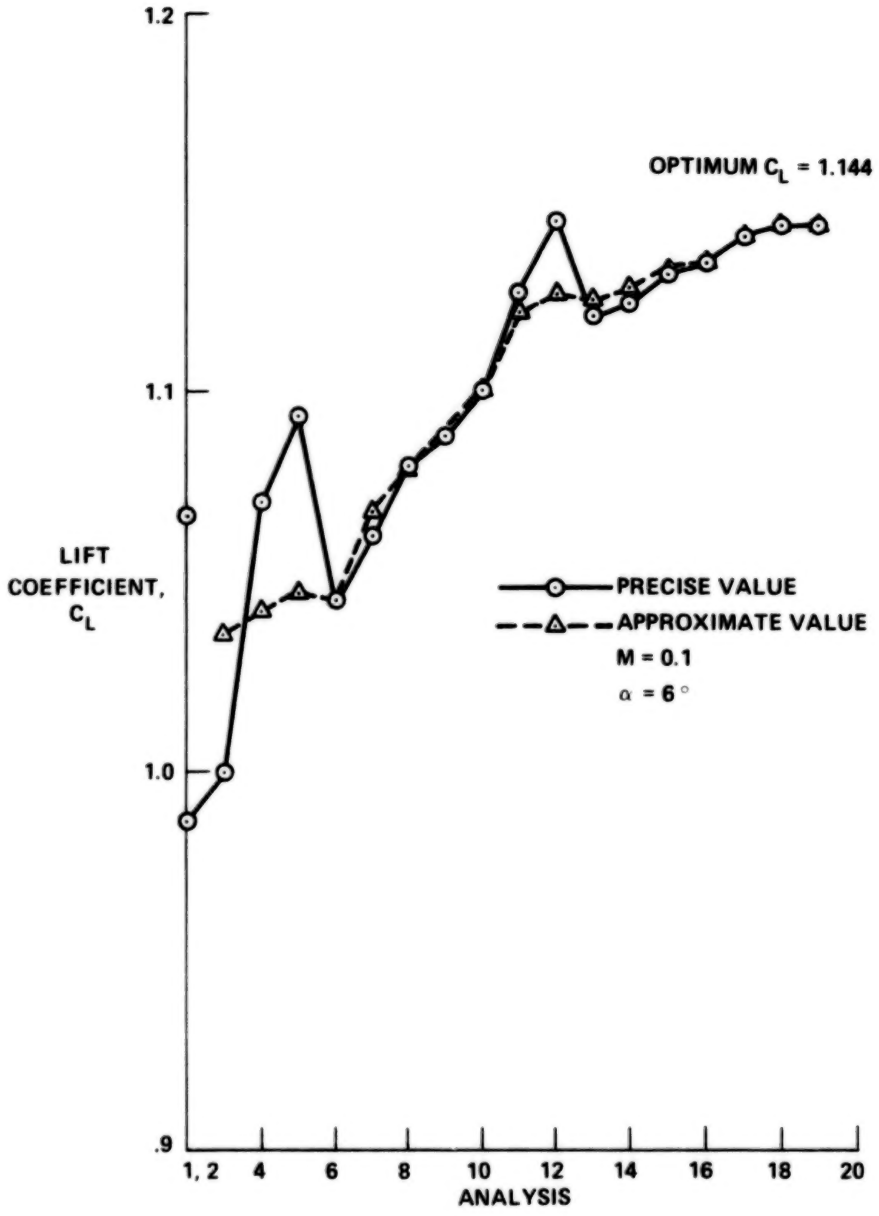


Figure 9.— Optimization history for example 1.

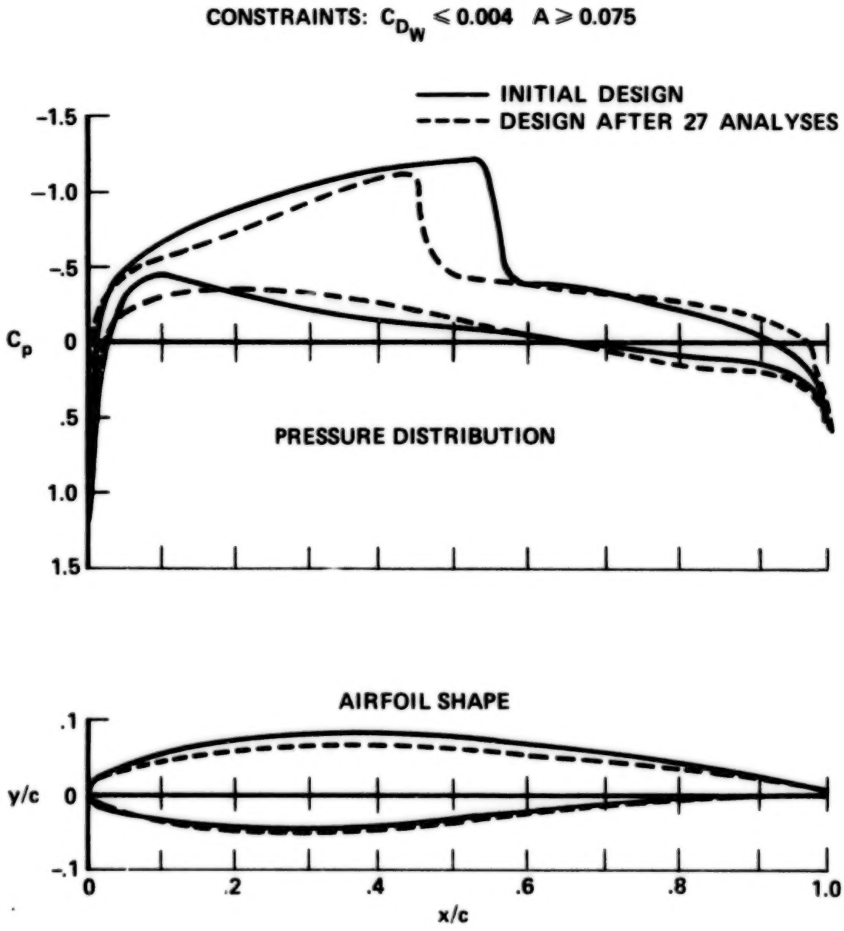


Figure 10.— Example 2A: Lift maximization, M = 0.75, α = 6°, 27 analyses.

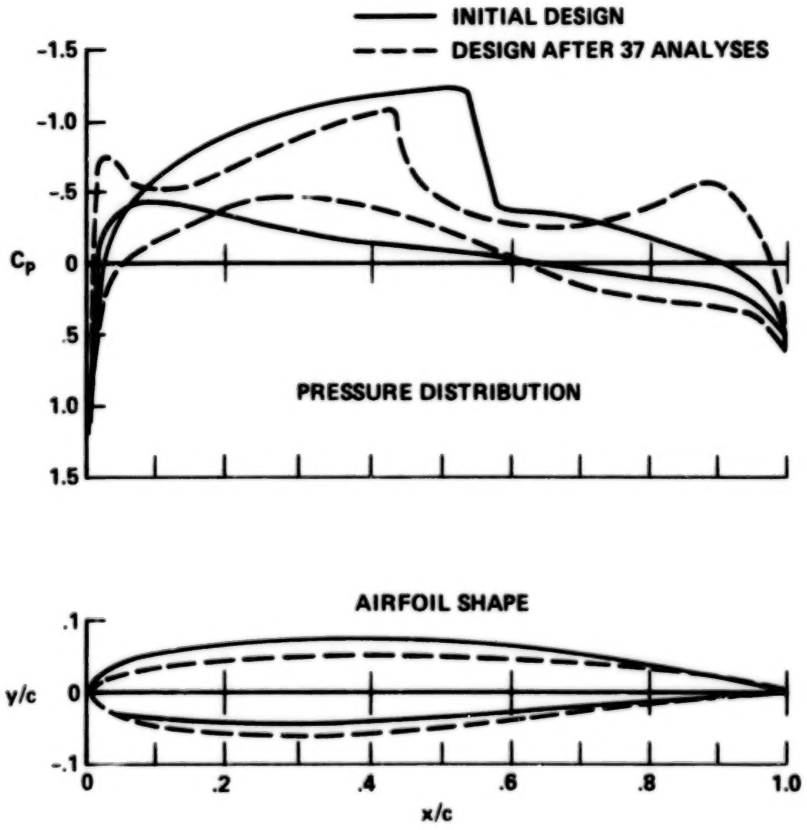


Figure 11.— Example 2B: Lift maximization, M = 0.75, α = 6°, 37 analyses.

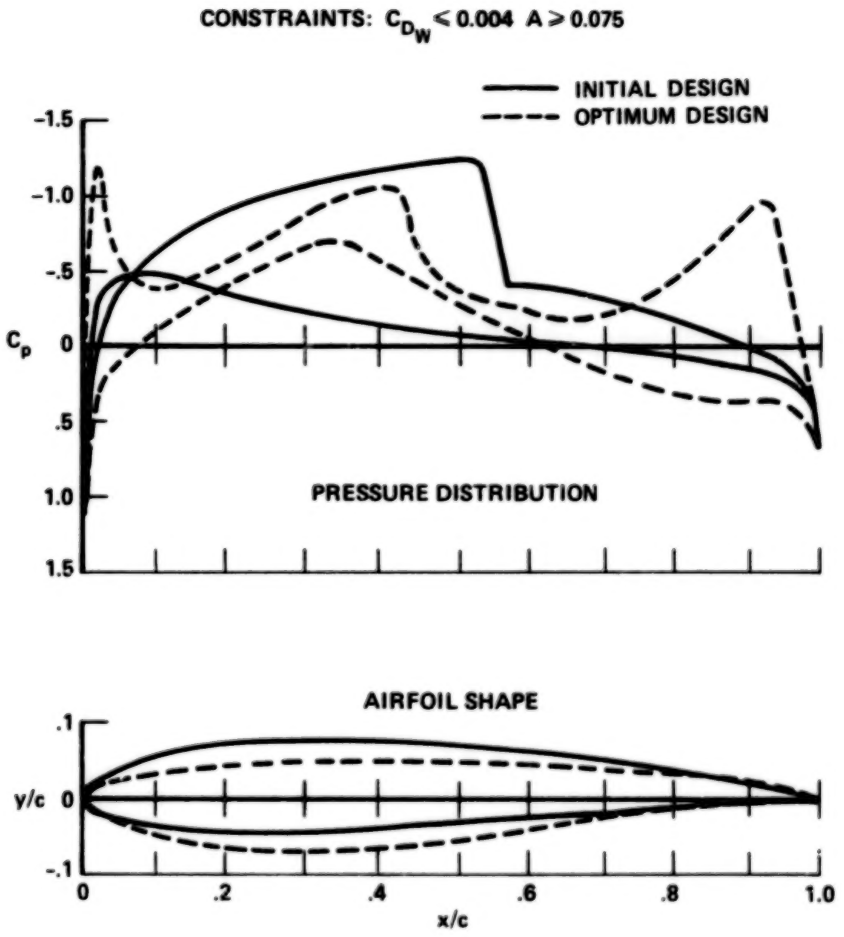


Figure 12.— Example 2C: Lift maximization, M = 0.75, α = 6°, 48 analyses.

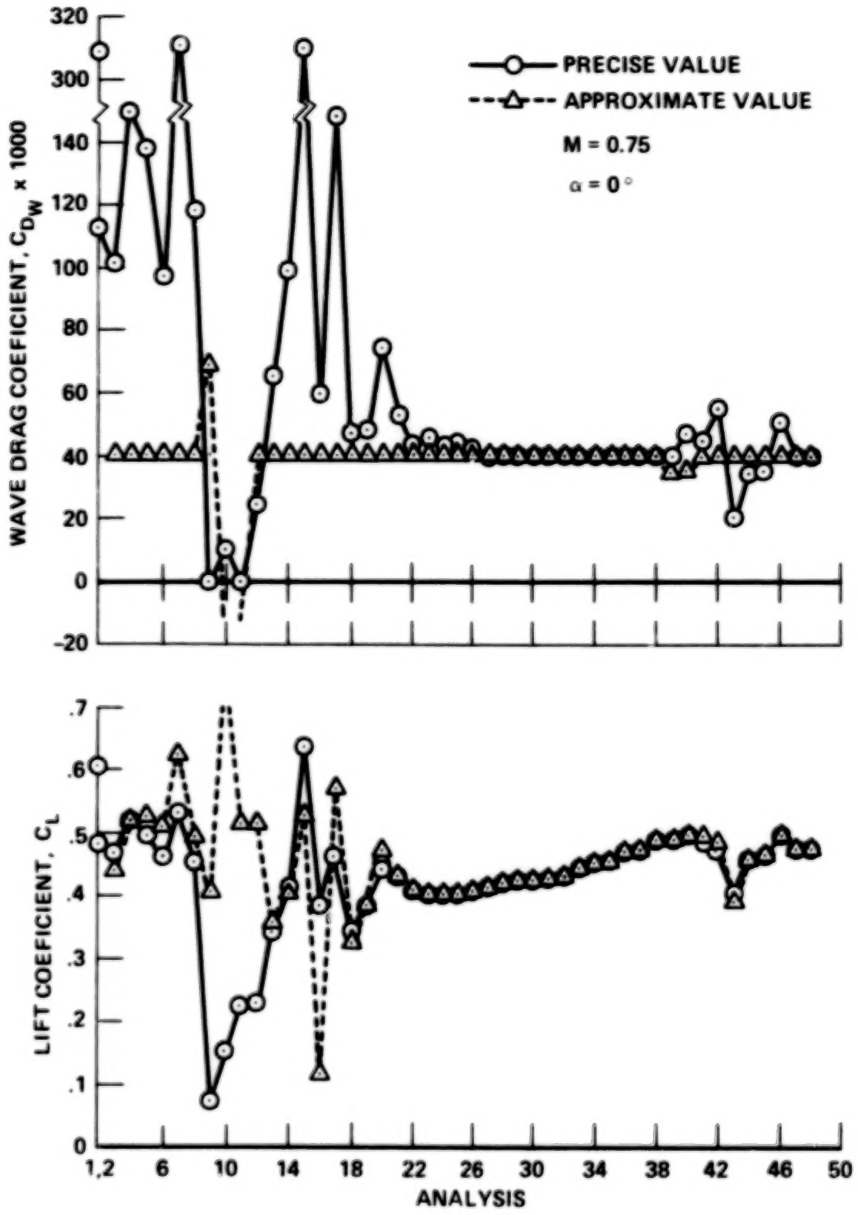


Figure 13.— Optimization history for example 2.

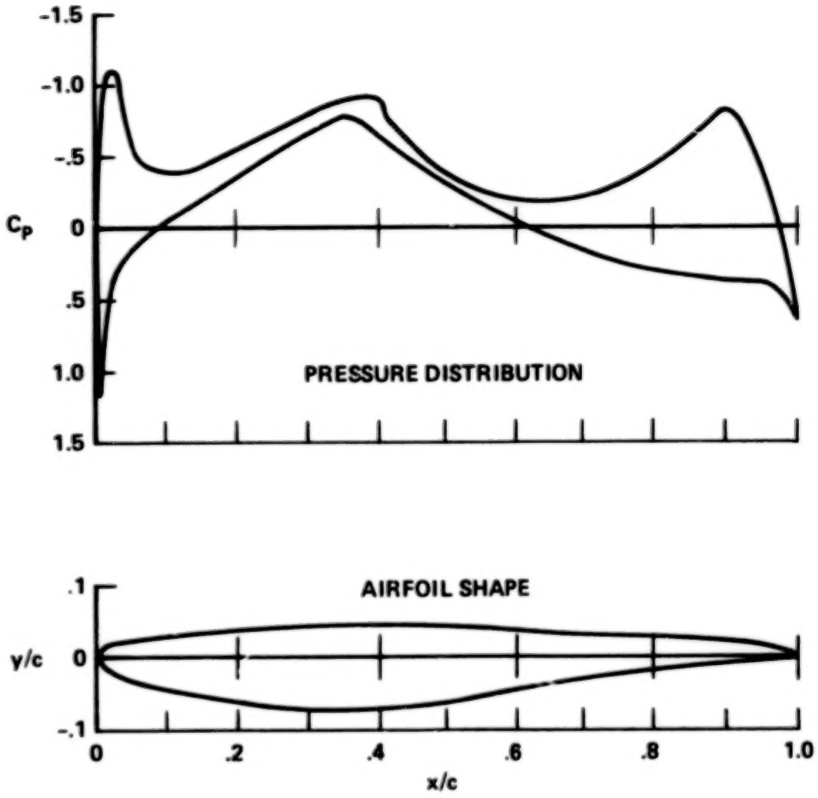


Figure 14.— Example 2D: Airfoil associated with analysis no. 43.

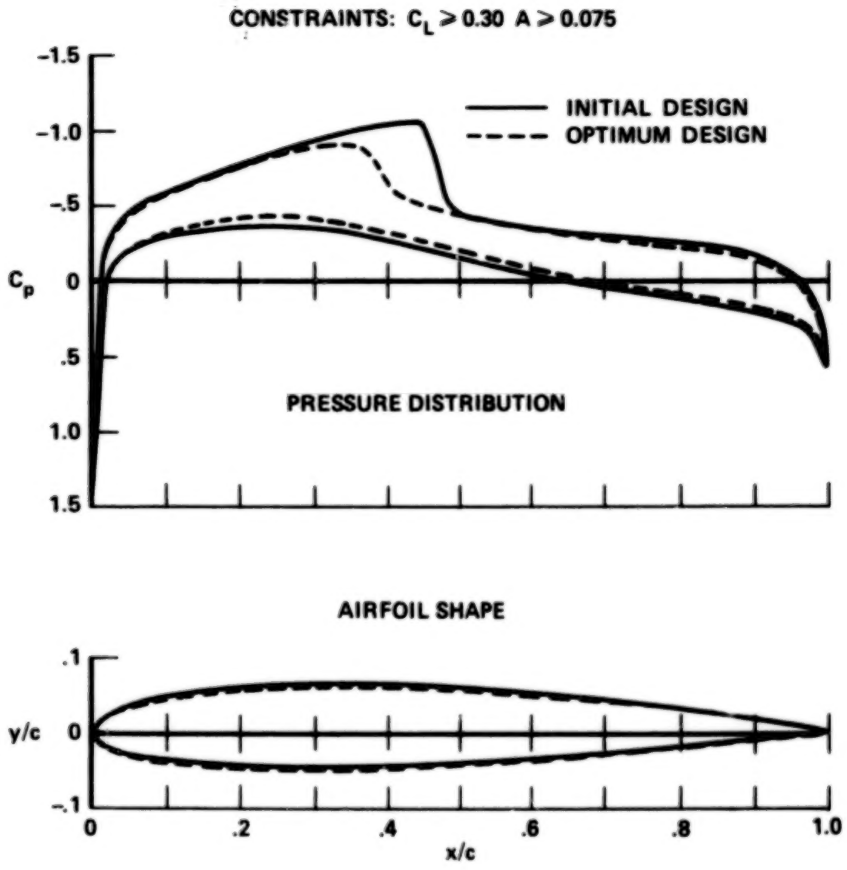


Figure 15.— Example 3A: Drag minimization, M = 0.75, α = 0°.

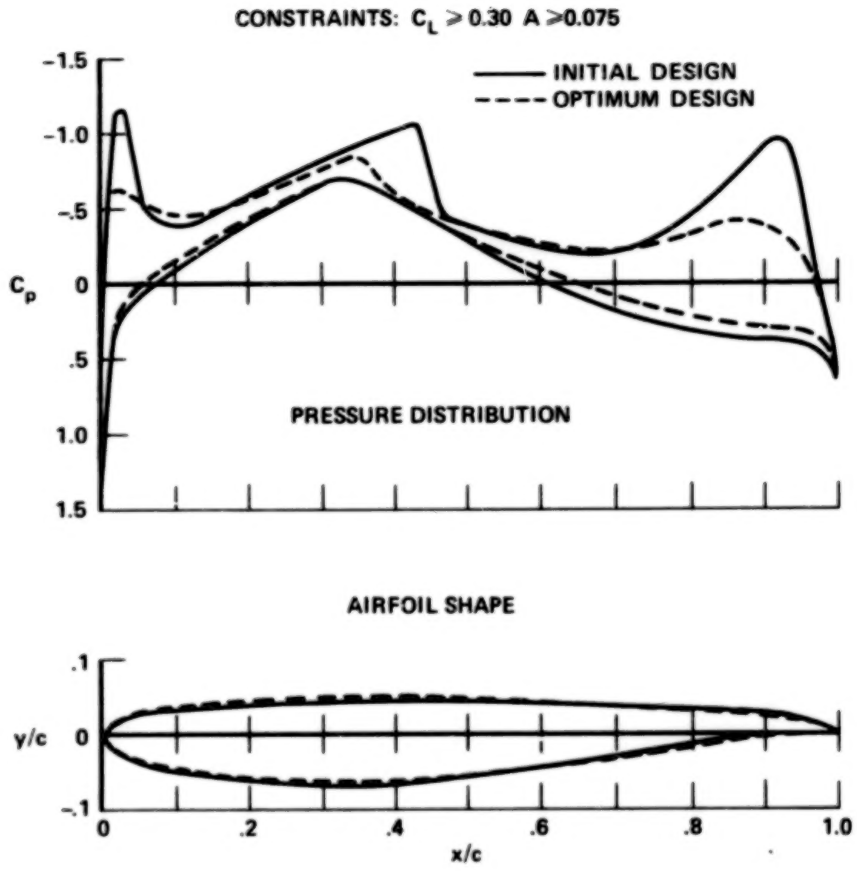


Figure 16.— Example 3B: Drag minimization, M = 0.75, $\alpha = 0°$.

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15. Supplementary Notes

16. Abstract

An efficient algorithm for airfoil optimization is presented. The algorithm utilizes approximation concepts to reduce the number of aerodynamic analyses required to reach the optimum design. Examples are presented and compared with previous results. Optimization efficiency improvements of more than a factor of 2 are demonstrated. Much greater improvements in efficiency are demonstrated when analysis data obtained in previous designs are utilized. The method is a general optimization procedure and is not limited to this application. The method is intended for application to a wide range of engineering design problems.

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